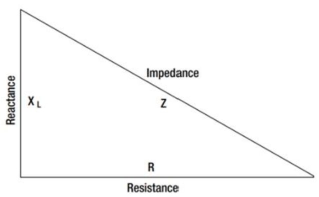
Resistive (R), Capacitive (C) and Inductive (L) circuits

Series AC Circuits

Impedance: may be illustrated by a right triangle.



$$Z^2 = R^2 + X_L^2$$

The square root of both sides of the equation gives:

$$Z = \sqrt{R^2 + X_L^2}$$

Resistance and reactance cannot be added directly, but they can be considered as two forces acting at right angles to each other

EX:

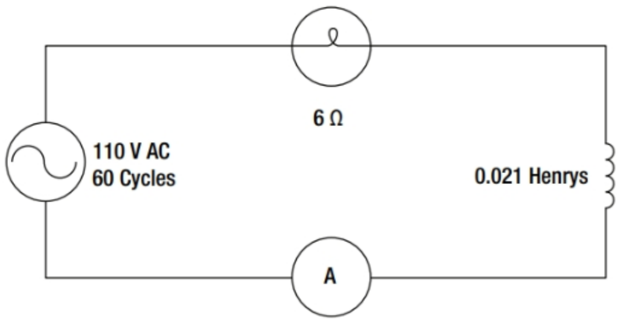


Figure 14-3. A circuit containing resistance and inductance.

$$X_L = 2\,\pi \times f \times L$$
$$X_L = 6.28 \times 60 \times 0.021$$
$$X_L = 8 \text{ ohms inductive reactance}$$

$$Z = \sqrt{R^2 + X_L^2}$$
$$Z = \sqrt{6^2 + 8^2}$$
$$Z = \sqrt{36 + 64}$$
$$Z = \sqrt{100}$$
$$Z = 10 \text{ ohms impedance}$$

Then the current flow,

$$I = \frac{E}{Z}$$
$$I = \frac{110}{10}$$

$$I = 11 \text{ amperes current}$$

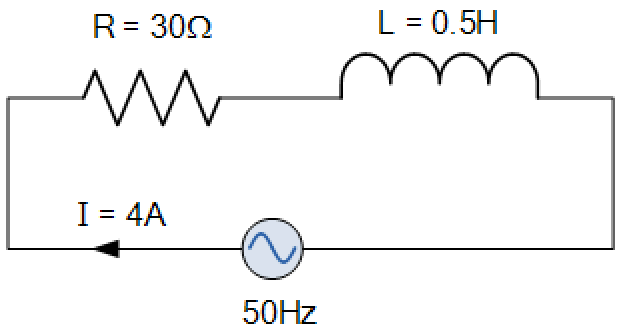
The voltage drop across the resistance (ER) is:

$$E^R = I \times R$$
$$E^R = 11 \times 6 = 66 \text{ volts}$$

The voltage drop across the inductance (E_{X_L}) is:

$$E_{X_L} = I \times X_L$$
$$E_{X_L} = 11 \times 8 = 88 \text{ volts}$$

Ex:



$$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157\Omega$$

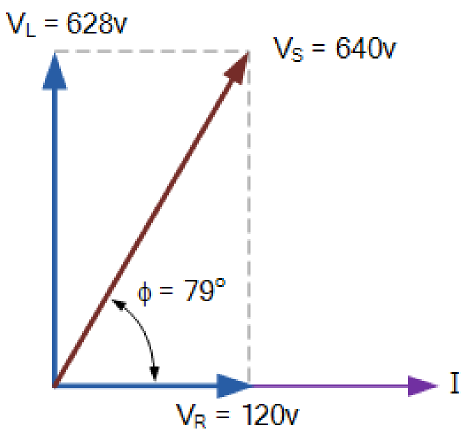
$$Z = \sqrt{R^2 + X_L^2}$$
$$Z = \sqrt{30^2 + 157^2}$$
$$Z = 159.8\Omega$$

$$V_S = I.Z = 4 \times 159.8 = 640v$$

$$V_R = I.R = 4 \times 30 = 120v$$

$$V_L = I.X_L = 4 \times 157 = 628v$$

$$\tan^{-1}\phi = \frac{X_L}{R} = \frac{157}{30} = 79.2^o$$ Phase angel



Ex: Rc

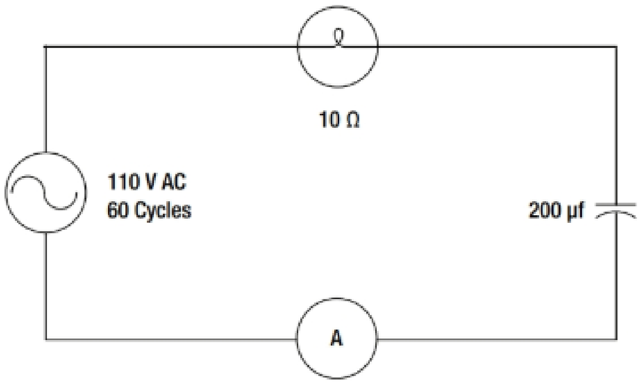


Figure 14-4. A circuit containing resistance and capacitance.

$$200\ \mu f = \frac{200}{1\ 000\ 000} = 0.000\ 200 \text{ farads}$$

$$X_C = \frac{1}{2\ \pi\ f\ C}$$

$$X_C = \frac{1}{6.28 \times 60 \times 0.002\ 00 \text{ farads}}$$

$$X_C = \frac{1}{0.075\ 36}$$

X_C = 13 ohms capacitive reactance

To find the impedance:

$$Z = \sqrt{R^2 + X_C{}^2}$$
$$Z = \sqrt{10^2 + 13^2}$$
$$Z = \sqrt{100 + 169}$$
$$Z = \sqrt{269}$$

Z = 16.4 ohms capacitive reactance

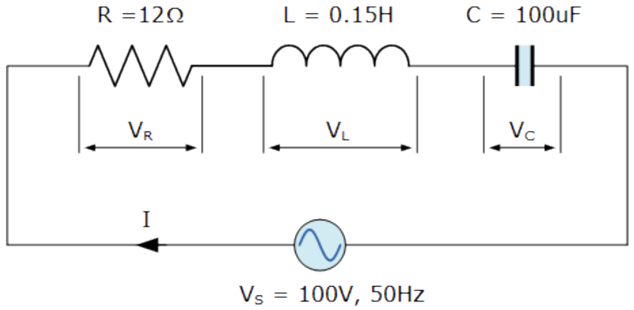
To find the current:

$$I = \frac{E}{Z}$$

$$I = \frac{110}{16.4}$$

I = 6.7 amperes

Ex: RLC



$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.13\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

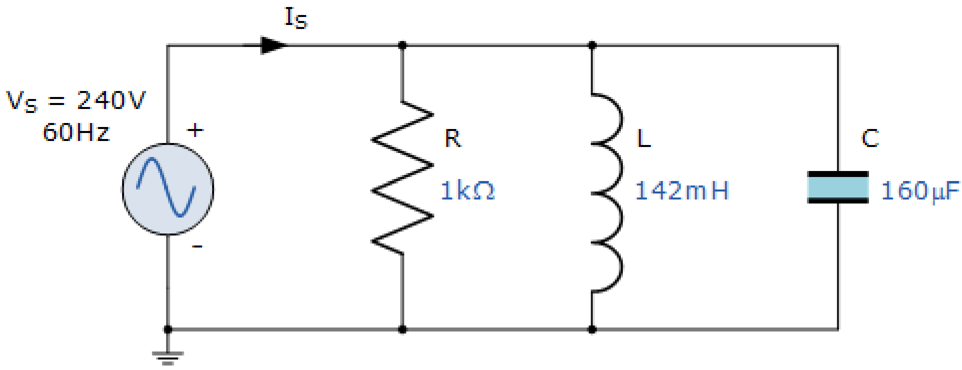
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega$$

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.14\,\text{Amps}$$

Parallel AC Circuits



$$X_L = \omega L = 2\pi f L = 2\pi .60.142\times 10^{-3} = 53.54\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi .60.160\times 10^{-6}} = 16.58\Omega$$

$$IL = \frac{V}{XL} \qquad IC = \frac{v}{XC}$$

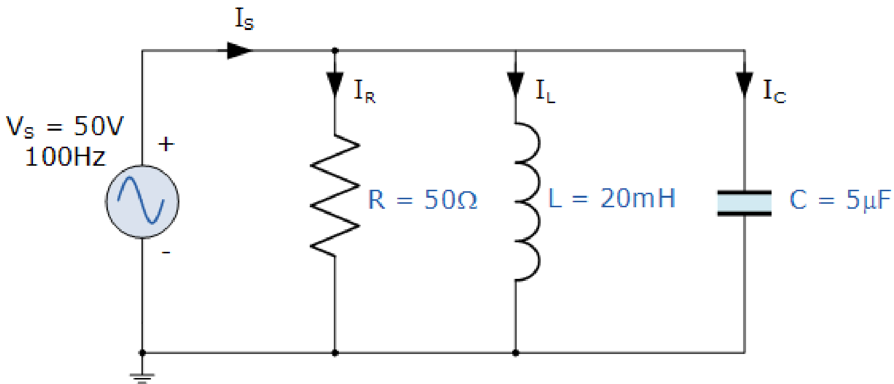
$$IL = \frac{240}{53.54} = 4.48A \qquad IC = \frac{240}{16.58} = 14.4A \qquad IR = \frac{240}{1000} = 0.24A$$

$$I = \sqrt{IR^2 + (IL - IC)^2} \qquad I = \sqrt{(0.24)^2 + (4.48 - 14.4)^2} = 9.9A$$

$$z = \frac{V}{I} = \frac{240}{9.9} = 24.24\Omega$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{1000}\right)^2 + \left(\frac{1}{53.54} - \frac{1}{16.58}\right)^2}}$$

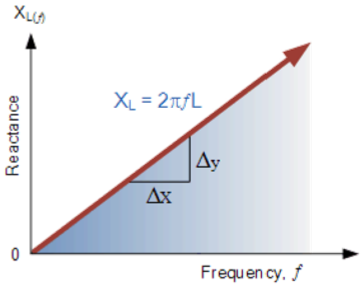
$$Z = \frac{1}{\sqrt{1.0\times 10^{-6} + 1.734\times 10^{-3}}} = \frac{1}{0.0417} = 24.0\Omega$$



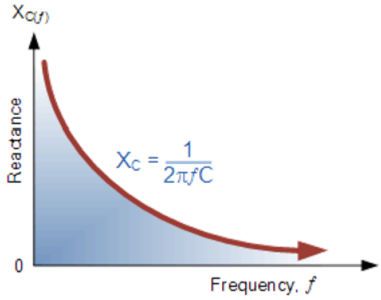
Series Resonance Circuit

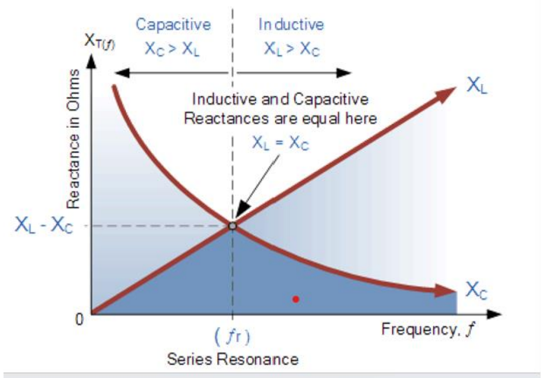
Resonance occurs in a series circuit when the supply frequency causes the voltages across L and C to be equal and opposite in phase.

Inductive Reactance against Frequency



Capacitive Reactance against Frequency





Resonance occurs in a series circuit when the supply frequency causes the voltages across L and C to be equal and opposite in phase.

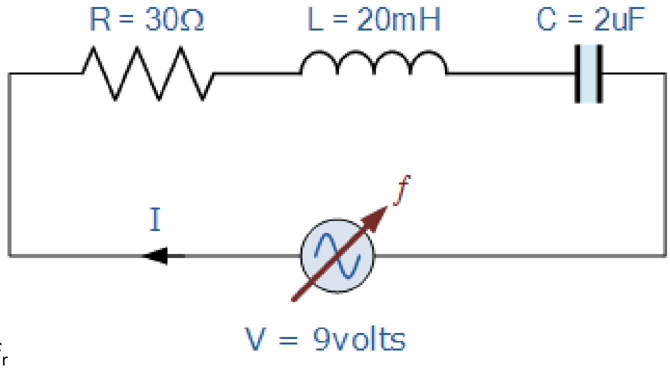
$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \ f_r = \frac{1}{2\pi\sqrt{LC}} \ (\text{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \ (\text{rads})$$

Ex:



Resonant Frequency, f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02\times2\times10^{-6}}} = 796\text{Hz}$$

Circuit Current at Resonance, I_m

$$I = \frac{V}{R} = \frac{9}{30} = 0.3\text{A or } 300\text{mA}$$

Inductive Reactance at Resonance, X_L

$$X_L = 2\pi fL = 2\pi\times796\times0.02 = 100\Omega$$

Voltages across the inductor and the capacitor, V_L, V_C

$$V_L = V_C$$
$$V_L = I\times X_L = 300\text{mA}\times100\Omega$$
$$V_L = 30\text{volts}$$

I did not calculate the XC because its resonance frequency.

Power in AC Circuits

true power: is the product of the volts and the amperes in the circuit.

True Power Defined

1, **The power dissipated in the resistance of a circuit, or the power actually used in the circuit.**

2, In an AC circuit, a voltmeter indicates the effective voltage and an ammeter indicates the effective current.

Apparent Power Defined

1, **It is the product of effective voltage times the effective current, expressed in volt-amperes.**

2, It must be multiplied by the power factor to obtain true power available.

Power in AC Circuits

the true power is less than the apparent power. When there is capacitance or inductance in the circuit, the current and voltage are not exactly in phase,

power factor

power factor: is The ratio of the true power to the apparent power

and is usually expressed in percent.

$$\text{Power Factor (PF)} = \frac{100 \times \text{Watts (True Power)}}{\text{Volts} \times \text{Amperes (Apparent Power)}}$$

P = true power $\quad P = I^2R \quad P = \frac{E^2}{R}$

Measured in units of ***Watts***

Q = reactive power $\quad Q = I^2X \quad Q = \frac{E^2}{X}$

Measured in units of ***Volt-Amps-Reactive (VAR)***

S = apparent power $\quad S = I^2Z \quad S = \frac{E^2}{Z} \quad S = IE$

Measured in units of ***Volt-Amps (VA)***

$$pr = \sqrt{pa^2 - pt^2}$$

Phase angel: formulas

$$\tan^{-1}\frac{x}{R} \quad or \quad \tan^{-1}\frac{VL - VC}{VR}$$

$$\cos^{-1}\frac{R}{Z} \quad or \quad \cos^{-1}\frac{VR}{VS}$$

$$\sin^{-1}\frac{X}{Z} \quad or \quad \sin^{-1}\frac{VL - VC}{VS}$$