

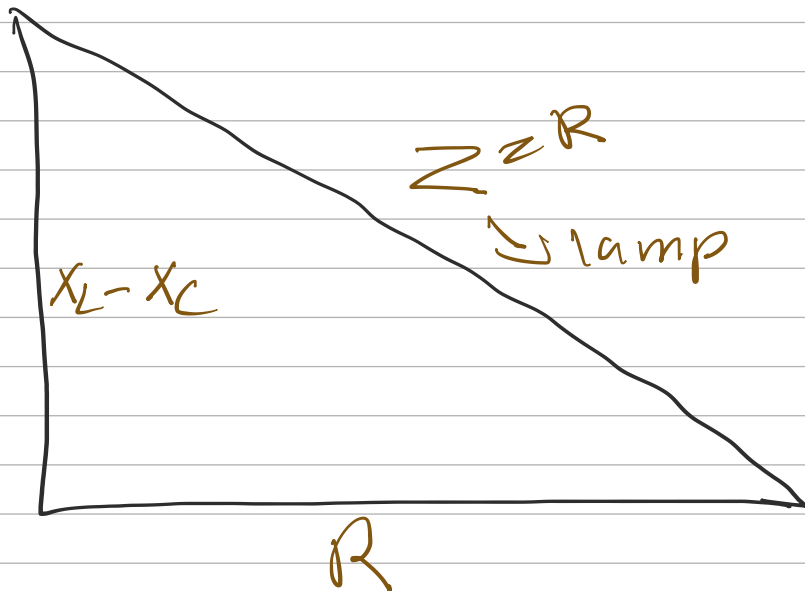
Series AC circuit:

if AC circuit consist of R only,

The value of impedance same as R

$$Z = R$$

$$I = \frac{V}{Z} \Rightarrow \text{instead of } R$$



if AC consist of R and either $[L]$ or $[C]$,

The Z not same as R $Z \neq R$

$$Z^2 = R^2 + X_L^2$$

$$Z = \sqrt{R^2 + \underbrace{(X_L - X_C)^2}_1}$$

دائماً ارقم البير
ناقص ارقم الفير
لعمري

Examples ٥٥

Resistive (R), Capacitive (C) and Inductive (L) Circuits

Resistors and Inductors (RL) in Series

Example: inductor

A series circuit consisting of 6Ω lamp and 0.021 H coil are connected in series to a source of 110 volts at 60 Hz . What is the value of the impedance and the current through the lamp and the coil?

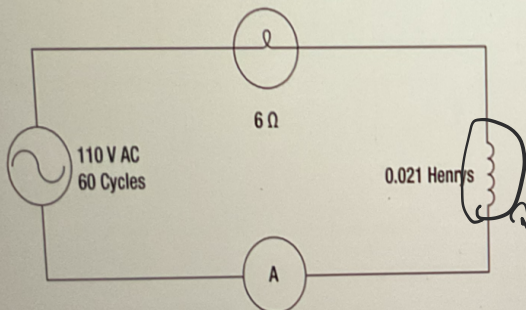


Figure 14-3. A circuit containing resistance and inductance.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_C = \frac{1}{2\pi F C}$$

$$X_L = 2\pi F L$$

نحن نستخدم X_L يعني

بطلع X_L بغير X_C

□ $R = 6\Omega$ lamp

$L = 0.02\text{ H}$ coil

$V_s = 110\text{ V}$

$F = 60\text{ Hz}$

$Z = ??$

$I = ??$

□ $X_L = 2\pi F L$

هنا بطلع X_L بغير X_C $= 6.28 \times 60 \times 0.021$

$= 7.9 \approx 8\Omega$

□ $Z = \sqrt{R^2 + X_L^2}$

unit (Ω) ohm $= \sqrt{6^2 + 8^2}$

$$= 10 \Omega$$

$$I = \frac{V}{Z}$$

unit
Ampere
(A)

$$= \frac{110}{10}$$

$$I = 11 \text{ A}$$

Resistive (R), Capacitive (C) and Inductive (L) Circuits

Resistors and Capacitors (RC) in Series

Example: capacitor

A series circuit in which a capacitor of $200 \mu\text{f}$ is connected in series with a 10 ohm lamp. What is the value of the impedance, the current flow, and the voltage drop across the lamp?

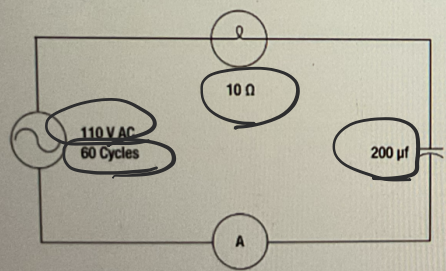


Figure 14-4. A circuit containing resistance and capacitance.

unit is micro
 $\times 10^{-6}$

$$C = 200 \mu\text{F} \text{ or } 200 \times 10^{-6} \text{ F}$$

$$X_C = \frac{1}{2\pi f C}$$

$$R = 10 \Omega \text{ lamp}$$

$$= \frac{1}{6.28 \times 60 \times 200 \times 10^{-6}}$$

$$f = 60 \text{ Hz}$$

$$= 13.3 \Omega$$

$$V = 110 \text{ V}$$

$$Z = ??$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$I = ??$$

$$= \sqrt{10^2 + 13.3^2}$$

$$V_{\text{drop}} = ??$$

$$= 16.6 \approx 17 \Omega$$

$$[3] \quad I = \frac{V}{Z}$$

$$= \frac{110}{17}$$

$$= 6.5 \approx 7 \text{ A}$$

$$[4] \quad V_{\text{drop}} = \underset{\substack{\downarrow \\ \text{total}}}{I} \times R_{\text{lamp}}$$

$$= 6.5 \times 10$$

$$= 65 \text{ V}$$

parallel AC circuit:

$$\frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{X_L^2} \quad \text{impedance}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

Current :

$$I_R = \frac{V}{R}$$

Voltage :

$$V_R = I R$$

$$I_C = \frac{V}{X_C}$$

$$V_C = I X_C$$

$$I_L = \frac{V}{X_L}$$

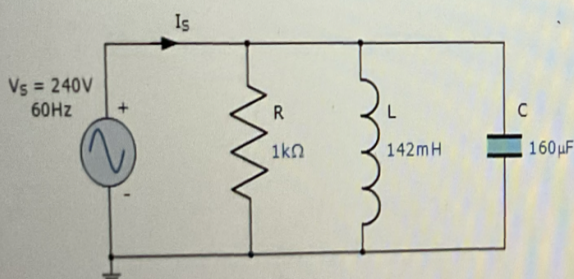
$$V_L = I X_L$$

Resistive (R), Capacitive (C) and Inductive (L) Circuits

RCL Parallel Circuit

Example:

A $1\text{k}\Omega$ resistor, a 142mH coil and a $160\mu\text{F}$ capacitor are all connected in parallel across a 240V , 60Hz supply. Calculate the impedance of the parallel RLC circuit and the current drawn from the supply.



$$R = 1k\Omega \text{ or } 1 \times 10^3 \Omega$$

$$L = 142mH \text{ or } 142 \times 10^{-3} H$$

$$C = 160\mu F \text{ or } 160 \times 10^{-6}$$

$$\boxed{1} \quad X_L = 2\pi FL$$

$$= 6.28 \times 60 \times 142 \times 10^{-3}$$

$$= 53.5 \Omega$$

$$X_C = \frac{1}{2\pi FC}$$

$$= \frac{1}{6.28 \times 60 \times 160 \times 10^{-6}}$$

$$= 16.6 \Omega$$

$$V = 240V$$

$$f = 60Hz$$

$$Z = ??$$

$$I = ??$$

من
parallel

الصيغة بحد

هو الرقم الكبير
والرقم منه

[larger - smaller.]

$$\boxed{2} \quad Z =$$

$$\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$= \sqrt{\left(\frac{1}{1 \times 10^3}\right)^2 + \left(\frac{1}{53.5} - \frac{1}{16.6}\right)^2}$$

$$= 24 \Omega$$

$$\boxed{3}$$

$$I = \frac{V}{Z}$$

$$= \frac{240}{24}$$

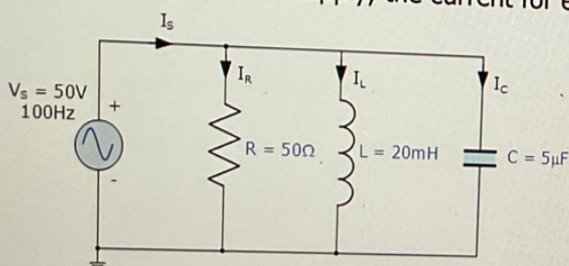
$$= 10A$$

Resistive (R), Capacitive (C) and Inductive (L) Circuits

RCL Parallel Circuit

Example:

A 50Ω resistor, a $20mH$ coil and a $5\mu F$ capacitor are all connected in parallel across a $50V$, $100Hz$ supply. Calculate the total current drawn from the supply, the current for each branch, the total impedance of the circuit.



$$R = 50 \Omega$$

$$L = 20 \text{ mH or } 20 \times 10^{-3} \text{ H}$$

$$C = 5 \mu\text{F or } 5 \times 10^{-6} \text{ F}$$

[1]

$$X_L = 2\pi fL$$

$$= 6.28 \times 100 \times 20 \times 10^{-3}$$

$$= 12.56 \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{6.28 \times 100 \times 5 \times 10^{-6}}$$

$$= 318.5 \Omega$$

$$V = 50 \text{ V}$$

$$f = 100 \text{ Hz}$$

[2]

$$Z =$$

$$\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{318.5} - \frac{1}{12.56}\right)^2}$$

$$= 12.7 \Omega$$

$$I_T = ??$$

find current of each branch

$$I_R = ??$$

$$I_L = ??$$

$$I_C = ??$$

$$Z_T = ??$$

[3]

$$I_T = \frac{V}{Z_T}$$

$$= \frac{50}{12.7}$$

$$= 3.9 \approx 4 \text{ A}$$

[4] $I_R = \frac{V}{R} \quad \left\{ \quad I_L = \frac{V}{X_L} \right.$

$$= \frac{50}{80}$$

$$I_R = 1 \text{ A}$$

$$= \frac{50}{12.56}$$

$$= 3.9 \text{ A}$$

$$I_C = \frac{V}{X_C}$$

$$= \frac{50}{318.5}$$

$$= 0.1 \text{ A}$$

parallel Resonance :-

in parallel resonance the
inductor reactance and

capacitor reactance are
equal $X_L = X_C$

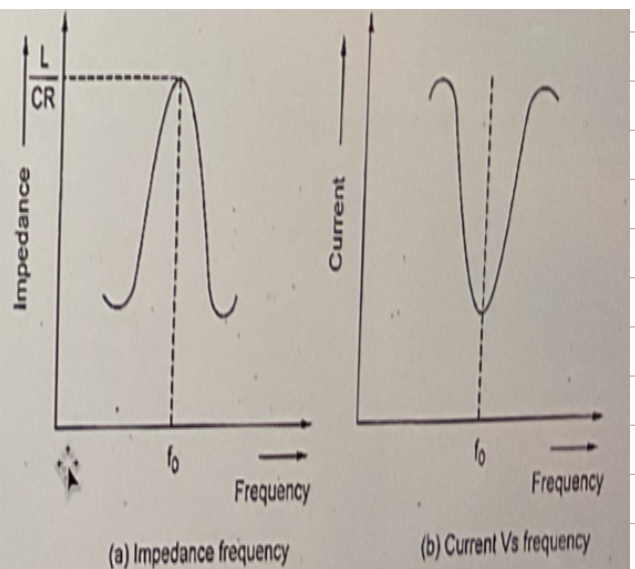
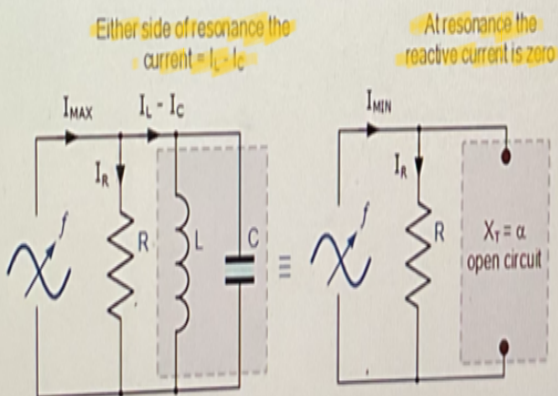
$$f = \frac{1}{2\pi \sqrt{LC}}$$

6.28 inductor capacitor

→ Circuit impedance Z is maximum

→ Circuit current I is minimum

Circuit current is minimum.

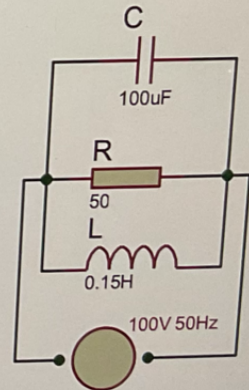


Parallel Resonance

Example:

A 50Ω resistor is placed in parallel with a 0.15H inductor and a $100\mu\text{F}$ capacitor with voltage supply of 100V , 50Hz . Calculate:

- Resonant frequency
- The current in each branch
- The supply current
- The phase angle between supply current and supply voltage



$$R = 50\Omega$$

$$L = 0.15\text{H}$$

$$C = 100\mu\text{F} \text{ or } 100 \times 10^{-6}$$

$$V = 100\text{V}$$

$$f = 50\text{Hz}$$

$$f_{\text{resonant}} = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{6.28 \times \sqrt{(0.15)(100 \times 10^{-6})}}$$

$$= 41.1\text{Hz}$$

[2]

$$f_{\text{resonant}} = ??$$

$$X_L = 2\pi fL$$

$$= 6.28 \times 50 \times 0.15$$

$$47.1\Omega$$

I in each branch??

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{6.28 \times 50 \times 100 \times 10^{-6}}$$

$$= 31.8\Omega$$

[3]

$$I_R = \frac{V}{R}$$

$$I_C = \frac{V}{X_C}$$

$$= \frac{100}{50}$$

$$= 2 \text{ A}$$

$$= \frac{100}{31.8}$$

$$= 3.1 \text{ A}$$

$$I_L = \frac{V}{X_L}$$

$$= \frac{100}{47.1}$$

$$= 2.1 \text{ A}$$

$$[4] Z = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{47.1} - \frac{1}{31.8}\right)^2}}$$

$$= 44.5 \text{ } \Omega$$

$$[5] I_T = \frac{V}{Z_T}$$

$$= \frac{100}{44.5}$$

$$= 2.2 \text{ A}$$

$$[6] \text{ phase angle} = ??$$

I_C

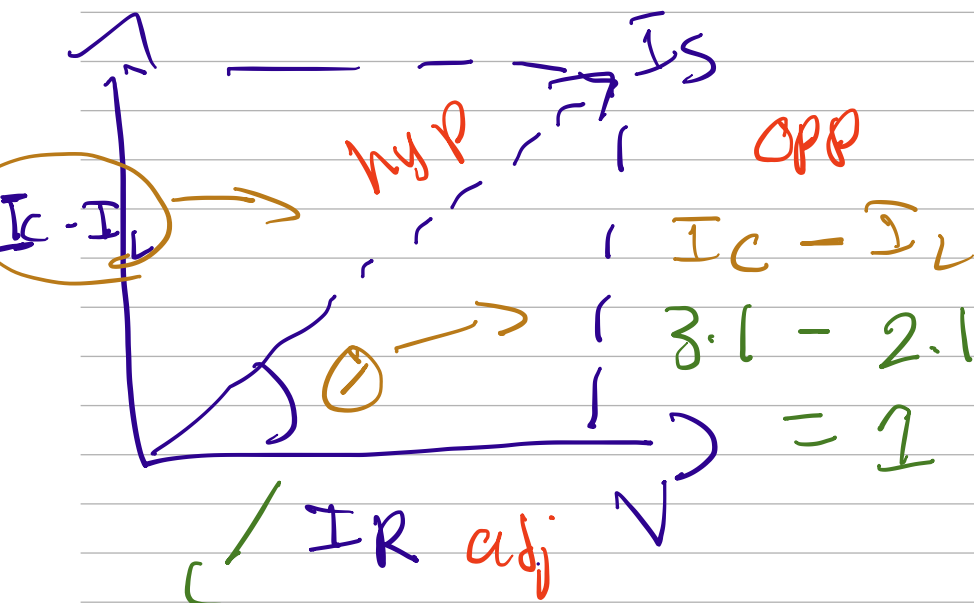
$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\theta = \tan^{-1} \left(\frac{1}{2} \right)$$

$$= 26.56^\circ$$

دایا اگر ازا کان parallel

تكون جی ارسه

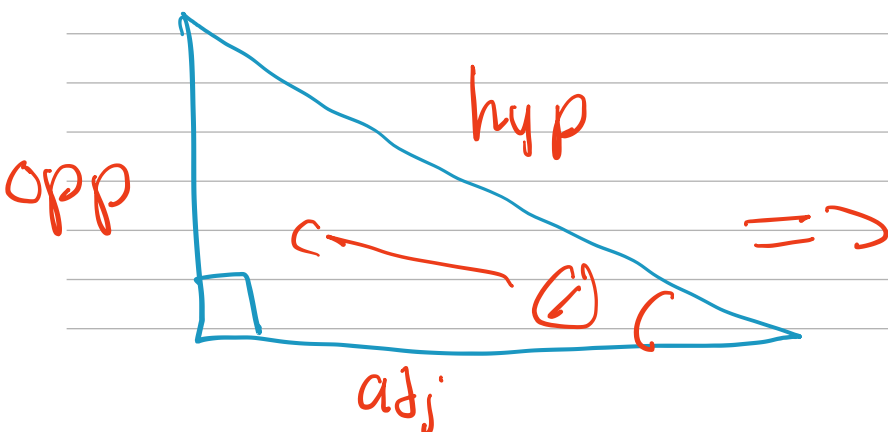


$$\frac{\text{opp}}{\text{adj}}$$

$$\tan^{-1} \left(\frac{1}{2} \right)$$

$$\theta = 26.56^\circ$$

انجوف ازا
angle و زاویه
opp



هنا ايسه
تستخرج ازا واحد
tan
يعني نصف الجواب
cos
دنا
جروا

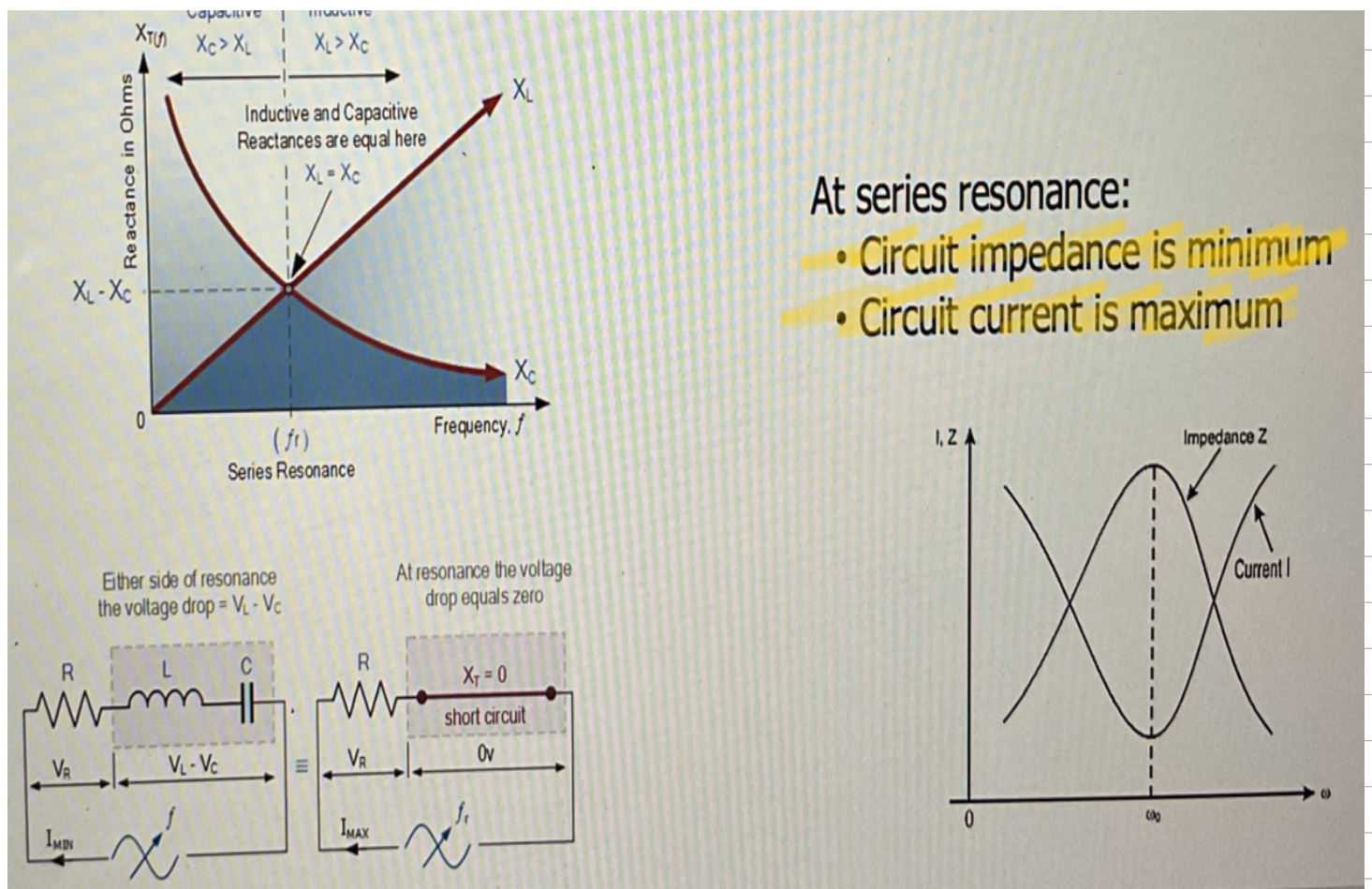
Series Resonant:

At series resonant the inductor and capacitor is equal $X_C = X_L$

→ circuit impedance Z is minimum

→ circuit current I is maximum.

$$f = \frac{1}{2\pi \sqrt{LC}}$$

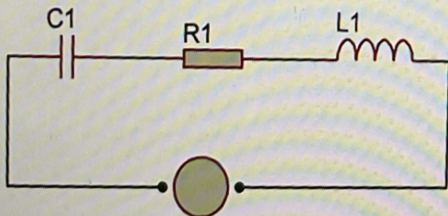


Series Resonance

Example:

In the following circuit at resonance, calculate: (1) The resonant frequency, (2) the current flowing in the circuit at resonance, (3) the voltage drop across each component at resonance.

[$C1 = 60 \mu F$; $R1 = 10 \Omega$; $L1 = 125 \text{ mH}$; Supply voltage = 120 V]



Resonant $f = ? ?$ ✓✓

I at resonant $= ? ?$ ✓✓

V drop across each component $= ? ?$

$$f_{\text{resonant}} = \frac{1}{2\pi \sqrt{LC}}$$

$$= \frac{1}{6.28 \times \sqrt{(125 \times 10^{-3}) \times (60 \times 10^{-6})}}$$

$$C_1 = 60 \mu F \text{ or } 60 \times 10^{-6} F$$

$$= 58.14 \text{ Hz}$$

$$R_1 = 10 \Omega$$

$$L_1 = 125 \text{ mH or } 125 \times 10^{-3} \text{ H}$$

$$V = 120 \text{ V}$$

$$\boxed{2} \quad X_L = 2\pi f L$$

$$= 6.28 \times 58.14 \times 125 \times 10^{-3}$$

$$= 45.63 \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{6.28 \times 58.14 \times 60 \times 10^{-6}}$$

$$= 45.64 \Omega$$

in series resonant the X_L and X_C are equal $X_L = X_C$

$$\boxed{3} \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$R = Z$$

$$= \sqrt{10^2 + (45.63 - 45.63)^2}$$

$$= 10 \Omega$$

$$\boxed{4} \quad I_T = \frac{V}{Z}$$

$$= \frac{120}{10}$$

$$= 12 \text{ A}$$

$$\boxed{5} \quad V_R = I R$$

$$= 12 \times 10$$

$$= 120 \text{ V}$$

$$V_C = I X_C$$

$$= 12 \times 45.64$$

$$= 547.68 \text{ V}$$

$$V_L = I X_L$$

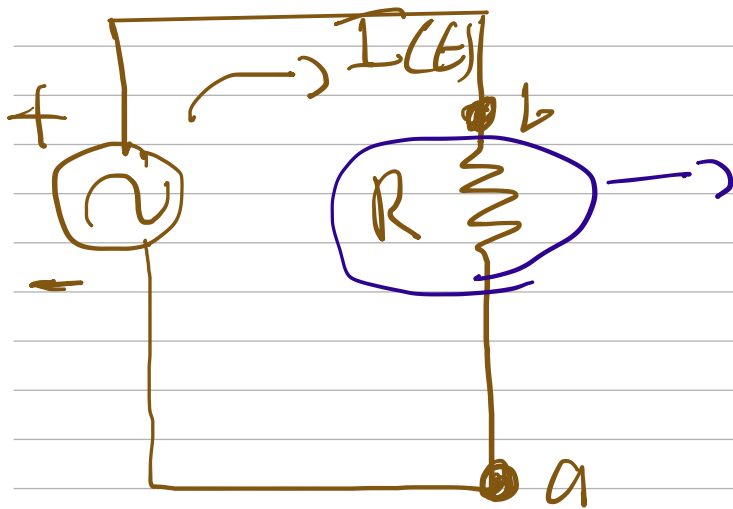
$$= 12 \times 45.63$$

$$= 547.56 \text{ V}$$

pure Resistor :-

$$R = \frac{V_{pk}}{I_{pk}}$$

$$R = \frac{V_{rms}}{I_{rms}}$$



↙ ↘
resistor

unit ohm (Ω)

inductors in AC circuits

unit ohm (Ω)

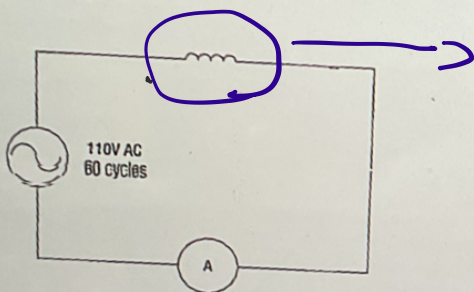
↙ ↘
 X_L

$$= 2\pi f L$$

↙ ↘
6.28 constant

Inductors in AC circuits

- AC series circuit is shown in which the inductance is 0.146 Henry, and the voltage is 110 volts at a frequency of 60 cycles per second. Determine inductive reactance?



↙ ↘
inductor

$$L = 0.146 \text{ H}$$

$$X_L = 2\pi FL$$

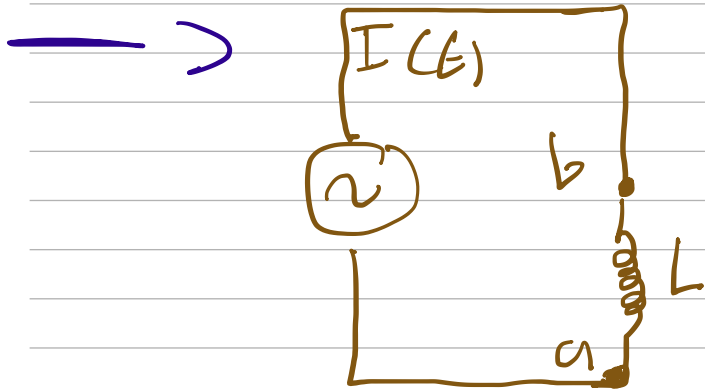
$$V = 110 \text{ V}$$

$$f = 60 \text{ cps}$$

$$= 6.28 \times 60 \times 0.146$$

$$X_L = ??$$

$$\approx 55.88$$



to find V_L :

$$V_L = I_L X_L$$

to find X_L

$$X_L = \frac{V_L}{I_L}$$

to find

$$I_L = \frac{V_L}{X_L}$$

* Voltage containing pure inductance will lead current by 90°

$L \Rightarrow$ unit Henry (H)

Capacitor in AC circuits:

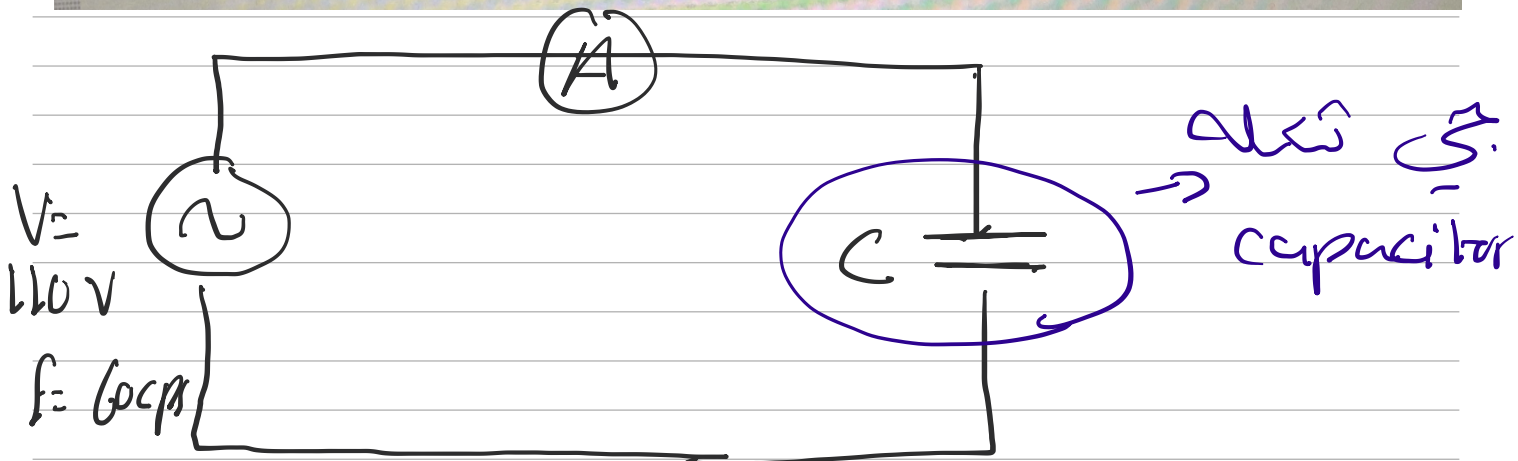
$$X_C = \frac{1}{2\pi FC}$$

unit ohm (Ω) unit Farad (F)

Capacitive Reactance

Problem:

A series circuit is assumed in which the impressed voltage is 110 volts at 60 cps, and the capacitance of a condenser is 80 μF . Find the capacitive reactance and the current flow.



$$V = 110 \text{ V}$$

$$f = 60 \text{ cps}$$

$$C = 80 \mu\text{F}$$

$$\text{or } 80 \times 10^{-6} \text{ F}$$

$$X_C = ??$$

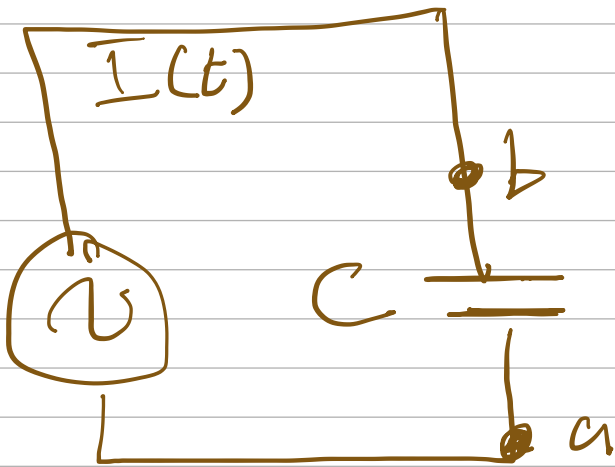
$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{6.28 \times 60 \times 80 \times 10^{-6}}$$

$$= 33.17 \Omega$$

→ voltage containing pure capacitor will lag current

by 90° .



to find V :

$$V_C = I X_C$$

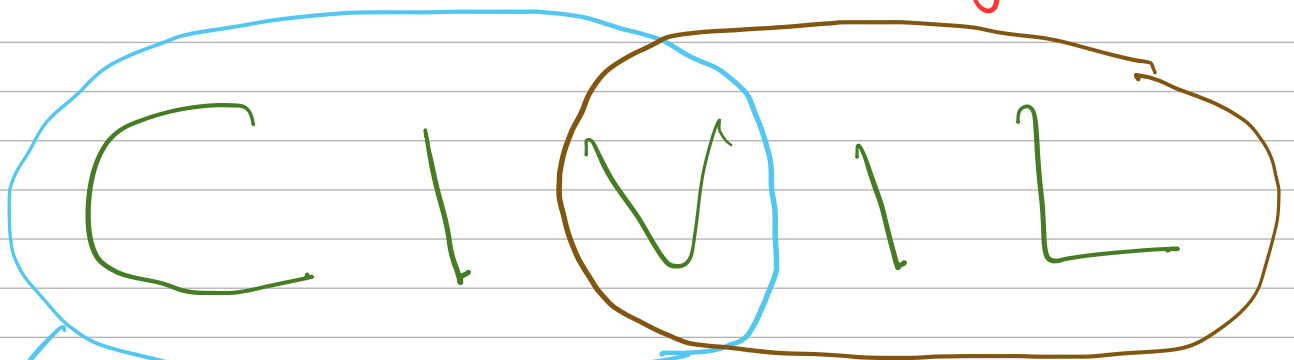
to find X_C :

$$X_C = \frac{V_C}{I}$$

to find I_C :

$$I_C = \frac{V_C}{X_C}$$

Phasor Diagrams:



in capacitor

in inductor,
current lag

Current and

Voltage.

Voltage

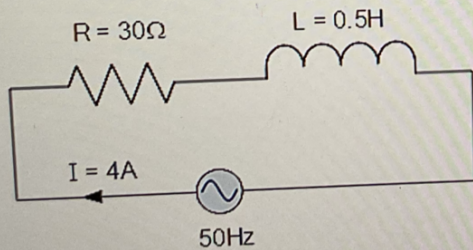
Resistors and Inductors (RL) in Series

Example:

RL Circuit

A coil has a resistance of 30Ω and an inductance of $0.5H$. If the current flowing through the coil is 4amps.

- What will be the value of the supply voltage if its frequency is $50Hz$?
- What will be the phase angle between the current and supply voltage?



$$X_L = 2\pi fL$$

$$I = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$\tan^{-1}\phi = \frac{X_L}{R}$$

$$[1] X_L = 2\pi fL$$

$$R = 30 \Omega$$

$$= 6.28 \times 50 \times 0.5$$

$$L = 0.5 H$$

$$= 157 \Omega$$

$$I = 4 A$$

$$[2] Z = \sqrt{R^2 + X_L^2}$$

$$V_s = ??$$

$$= \sqrt{30^2 + 157^2}$$

$$f = 50 Hz$$

$$= 159.84 \Omega$$

$$\phi = ??$$

between I and
supply V

$$[3] I = \frac{V}{Z} \Rightarrow V = IZ$$

$$= 4 \times 159.84$$

$$= 639.36 V$$

$$[4] \phi = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$\tan^{-1} \left(\frac{157}{30} \right)$$

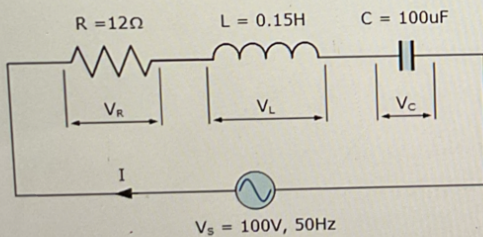
$$= 79.18^\circ$$

RLC Circuit

RLC Series Circuit

Example:

A series RLC circuit containing a resistance of 12Ω , an inductance of 0.15H and a capacitor of $100\mu\text{F}$ are connected in series across a 100V , 50Hz supply. Calculate the total circuit impedance and the circuit's current.



$$R = 12\Omega$$

$$L = 0.15\text{H}$$

$$C = 100\mu\text{F}$$

$$V = 100\text{V}$$

$$f = 50\text{Hz}$$

$$Z_T = ??$$

$$I_C = ??$$

$$\begin{aligned} \text{1} \quad X_C &= \frac{1}{2\pi f C} \\ &= \frac{1}{6.28 \times 50 \times 100 \times 10^{-6}} \\ &= 31.85\Omega \end{aligned}$$

$$\begin{aligned} X_L &= 2\pi f L \\ &= 6.28 \times 50 \times 0.15 \\ &= 47.1\Omega \end{aligned}$$

$$\begin{aligned} \text{2} \quad Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{12^2 + (47.1 - 31.85)^2} \\ &= 19.4\Omega \end{aligned}$$

$$\begin{aligned} \text{3} \quad I &= \frac{V}{Z} \\ &= \frac{100}{19.4} \\ &= 5.15\text{A} \end{aligned}$$

Power in AC circuit

Active power (P)
(Real power or True power)

$$P = I^2 R \quad P = \frac{V^2}{R}$$

obtained by "Wattmeter"

true ~~existing~~ R circuit ~~is~~ is
power

Reactive power (Q)

$$Q = I^2 X \quad Q = \frac{V^2}{X}$$

← reactance

X_L and X_C

it measured in **VAR**

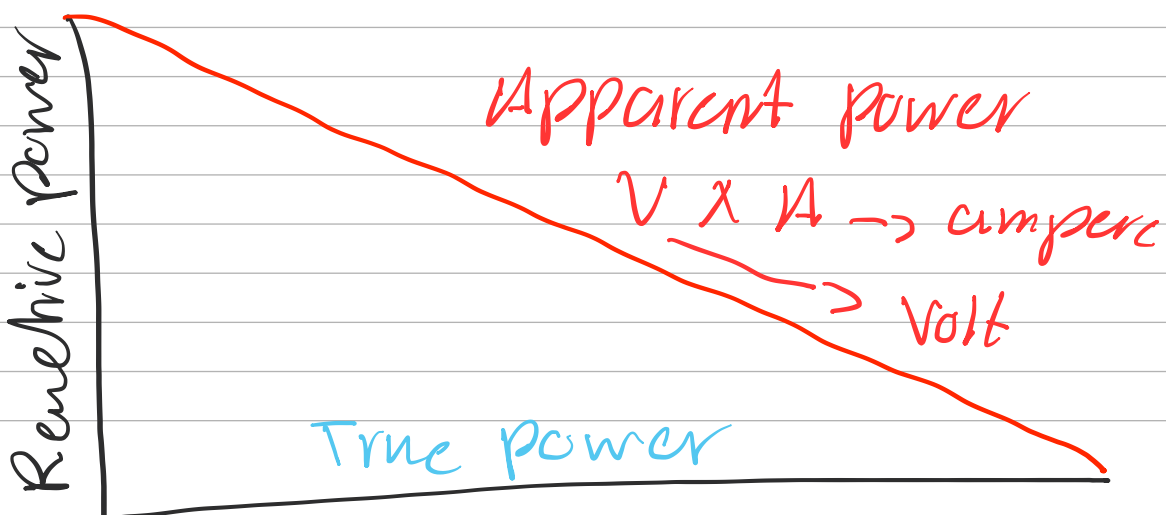
(volt-ampere-reactive)

reactive power is X circuit is X_L and X_C

Apparent power (S)

$$S = I^2 Z \quad S = \frac{V^2}{Z} \quad S = I V$$

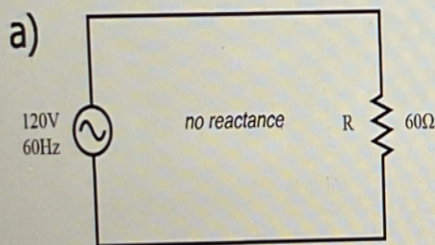
measured in **VA** (volt-ampere)



Examples

Example:

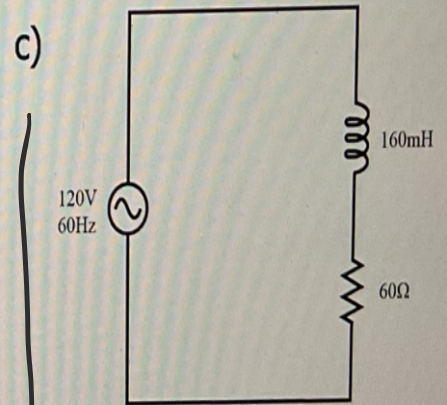
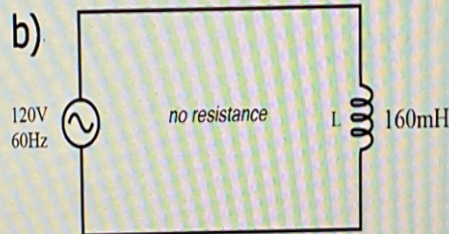
Find the true power, reactive power and apparent power for below circuits:



True power

$$Q = 0$$

$$P = V^2 / R$$



R has only
true power

1) True power:

$$V = 120V$$

$$f = 60Hz$$

$$R = 60\Omega$$

b) L has only
Reactive power

1) True power

R has only true power

$$P = 0$$

2) Reactive power

$$X_L = 2\pi fL$$

$$= 6.28 \times 60 \times 160 \times 10^{-3}$$

$$= 60.28V$$

2

c) R and X

1) True power

$$V = 120 \quad P = I^2 R$$

$$f = 60$$

$$R = 60 \quad I = \frac{V}{R} = \frac{120}{60}$$

$$L = 160 \times 10^{-3}$$

$$I = 2A$$

$$P = I^2 R$$

$$= 2^2 \times 60$$

$$= 240W$$

2) Reactive power

$$P = I^2 R$$

$$I = \frac{V}{R} = \frac{120}{60}$$

$$I = 2A$$

$$P = I^2 R = 2^2 \times 60 = 240W$$

[2] Reactive power

There is no reactive power.

لا توجد قوة تفاعلية
Reactive power

True power = S → Apparent power

$$Q = \frac{V^2}{X} \rightarrow X_L = \frac{120^2}{60.28}$$

$$= 238.88 VAR$$

[3] Apparent power

طاقة حقيقية P
أو قوة تفاعلية Q
يعني S

$$S = Q$$

$$Q = \frac{V^2}{X}$$

$$X_L = 2\pi FL = 6.28 \times 60 \times 160 \times 10^3$$

$$= 60.28 \Omega$$

$$Q = \frac{V^2}{X} = \frac{120^2}{60.28}$$

$$= 238.88 VAR$$

[3] Apparent power

$$S = \sqrt{P^2 + Q^2}$$

$$= \sqrt{240^2 + 238.88^2}$$

$$= 338.6 VA$$

Power Factor

P → true power

$$PF = \frac{P}{S} \times 100$$

S → Apparent power

Power Factor

Example:

A 220-volt AC motor takes 50 amperes from the line, but a wattmeter in the line shows that only 9350 watts are taken by the motor. What are the apparent power and the power factor?

$$V = 220V$$

$$I = 50A$$

$$P = 9350W$$

↓ True power
(wattmeter)

[1]

$$S = I V$$

$$= 50 \times 220$$

$$= 11000 VA$$

$$S = ??$$

Apparent
power

$$PF = ??$$

[2]

$$PF = \frac{P}{S} \times 100$$

$$= \frac{9350}{11000} \times 100$$

$$= 85\%$$

Useful Formulas:

$$\text{Capacitive Reactance: } X_C = \frac{1}{2\pi f C}$$

$$\text{Inductive Reactance: } X_L = 2\pi f L$$

$$\text{Impedance (series): } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{Impedance (parallel): } \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$\text{Resonance Frequency: } f_R = \frac{1}{2\pi\sqrt{LC}}$$

$$\text{True Power: } P = I^2 R$$

$$\text{Reactive Power: } P_R = I^2 X$$

$$\text{Apparent Power: } P_A = I^2 Z$$

$$\text{Cut-off Frequency: } f_C = \frac{1}{2\pi RC}$$

Nonfi Albetwanni